

# Premium Margins and Managing the Catastrophe Risk

Stephen Mildenhall  
CNA Insurance Companies  
August 22, 1995

## Introduction

Premium margins for a line of business are usually determined in two steps: first allocate surplus to the line and second compute a suitable return on the allocated surplus. It is then easy to compute a premium margin. Actuaries have suggested many different ways of computing a return on surplus: CAPM, return earned by industries of similar risk, options pricing theory, and so on. Also there have been various methods proposed to allocate surplus to lines of business: probability of ruin, expected policy-holder deficit, 2:1 premium to surplus, and so on. Papers on pricing usually concentrate either on surplus allocation or on rates of return but not both.

Since it is widely agreed that it is impossible to consistently allocate surplus to lines, the need for a surplus allocation is the Achilles heel of all current premium margin calculations, see [1]. This paper proposes a method to compute a premium margin *independent of the surplus allocated to the line of business!* The argument is that a lower surplus allocation results in a riskier line which must be compensated by a higher rate of return. Because this higher return is earned on a smaller amount of capital, the profit dollars per dollar of loss remains constant. This method can be thought of as allocating the profit dollars to lines of business. Heuristically, a smaller surplus allocation to a given line justifies in a higher target return because the line is more likely to exhaust its surplus allocation and have to "borrow" from other line's allocations. The higher return is required to pay for this increased claim on the other line's allocations.

The method proposed is closely related to CAPM. It derives a measure of risk from the correlation between the return on a given line and the return for the whole corporation. CAPM requires that only systematic risk be compensated, and, in analogy, we require that a line earns a higher rate of return only if it adds to the corporation's risk by having a return correlated with the total return. Insurance is predicated on a sharing of risk between independent insureds, so measuring risk through correlation between risks captures the quintessence of insurance. This measure of risk penalizes lines of business which behave as a group, for example homeowners insurance in the Northeast, as well as lines with a high variance, such as large catastrophe risks. The idea of using such a measure to compute rates of return was first proposed in [2].

This paper does not claim to compute a target return on equity for the whole corporation. Much has been written about this, see [2] for example. In particular, we are not claiming that a decrease in corporate surplus results in a higher target return. The amount of loss over premium and surplus allocation is important at the line level but not at the corporate level; this is an important difference between setting return targets for lines and for the whole corporation. A corporate return on equity target is in fact the only *variable* of the model presented here.

Nor does this paper discuss book-keeping issues of exactly how to measure profit, in terms of cash flows, taxes, discount factors, loss development etc. See [3] for six different techniques for computing profit and loading a given profit margin into the rates.

In the rest of the paper we describe the calculation and give two examples of its computed premium margins. Finally we show how the measure can be used as an effective cat management tool, giving branch and field offices different options in handling their cat exposures.

Throughout this paper "line" means any group of policies for which it is necessary to determine a target premium margin. For example, line could mean countrywide homeowners, private passenger auto liability in Pennsylvania, Specialty Ops SBU, or a single large commercial policy. In equations,  $EX$  denotes the expected value of a random variable  $X$ .

### Premium Margins

A premium margin is a sales margin on loss and expense dollars. It is not the same as a rate of return, which is a return on equity or allocated surplus. An insurance company may have a corporate rate of return on surplus goal of 15%. Additionally its individual SBUs (Strategic Business Units) may have premium margin goals expressed through target NEV (Net Economic Value) ranges. The purpose of this paper is to present a method of going from the 15% ROE goal to target NEV ranges.

In analogy with CAPM, we define the beta of the return on the line with the return for the entire corporation as a measure of the systematic risk of a line; symbolically:

$$\beta_l = \text{Cov}(r_l, r) / \text{Var}(r)$$

where

$$r_l = (P_l - L_l - E_l + r_f S_l) / S_l$$

is the return on the line  $l$ , and

$$r = (P - L - E + r_f S) / S$$

is the return for the whole corporation. Here a subscript  $l$  denotes line,  $P$  denotes premium,  $S$  denotes surplus,  $E$  denotes expenses,  $L$  denotes losses,  $r$  denotes return on surplus, and  $r_f$  denotes the risk free rate of interest.  $S_l$  represents any allocation of surplus to line  $l$ . It is not necessary that these allocations add up to the total surplus  $S$  available to the company. By convention, all loss sensitive elements are included with losses, so  $L$ ,  $L_l$  and hence  $r$ ,  $r_l$  are the only random variables in the above equations. This means that retro premiums are netted out of the losses rather than being added to the premium. Losses, expenses and premiums are all expressed as present values, and so include investment income. The term  $r_f S$  represents investment income on the surplus. As Miccolis points out in [4] this is no restriction if we measure return on a risk adjusted basis, since investing surplus in risk free instruments results in a lower target return  $Er$ . Substituting into the formula for beta we get:

$$\begin{aligned}
\beta_l &= \text{Cov}(r_l, r) / \text{Var}(r) \\
&= \frac{\text{Cov}((P_l - L_l - E_l + r_f S_l) / S_l, (P - L - E + r_f S) / S)}{\text{Var}((P - L - E + r_f S) / S)} \\
&= \frac{\text{Cov}(-L_l / S_l, -L / S)}{\text{Var}(-L / S)} \\
&= \frac{S}{S_l} \frac{\text{Cov}(L_l, L)}{\text{Var}(L)} \\
&= \frac{S}{S_l} \beta_l^*
\end{aligned}$$

where the last term defines beta star. Our assumption that losses are the only stochastic variable is used to go from the second to third line. Given beta for a line we define the target return on surplus for line  $l$  to be

$$E r_l = r_f + \beta_l (E r - r_f).$$

Notice that if  $l$  is the whole corporation,  $\beta_l = 1$  and  $r_x = r$  just as it should. This reflects the fact that our method says nothing about the target return for the whole corporation.

We suppose that a corporate ROE goal  $E r$  has been set, that total surplus available to the company is  $S$  and that an expected loss and expense dollar amount  $P_l$  is known by line. This information means that the corporation must earn  $E r \cdot S$  profit dollars, or

$$\Pi = (E r - r_f) S$$

dollars from insurance operations net of investment income earned on surplus. The problem is how to allocate these dollars to lines  $l$ . Using the target return on surplus and an arbitrary surplus allocation  $S_l$  to lines we compute the target premium margin as

$$\begin{aligned}
\pi_l &= (Er_l - r_f)S_l / P_l \\
&= \beta_l(Er - r_f)S_l / P_l \\
&= \frac{S_l}{P_l} \frac{S}{S_l} \beta_l^* (Er - r_f) \\
&= \frac{S}{P_l} \beta_l^* (Er - r_f) \\
&= \frac{\beta_l^*}{P_l} \Pi.
\end{aligned}$$

This expression is independent of the surplus allocation! The last line makes it clear how the method can be thought of as allocating profit dollars. A gross premium can be computed as

$$G_l = (1 + \pi_l)P_l$$

which can be converted into a desired operating target. An expense ratio approach can also be used to pay commission and expenses on the profit margin, or any other desired combination.

If we compute margins is this way, then the total profit earned by the corporation will be

$$\begin{aligned}
\sum_l P_l \pi_l &= \sum_l P_l \frac{S}{P_l} \beta_l^* (Er - r_f) \\
&= S(Er - r_f) \sum_l \beta_l^* \\
&= S(Er - r_f) \\
&= \Pi
\end{aligned}$$

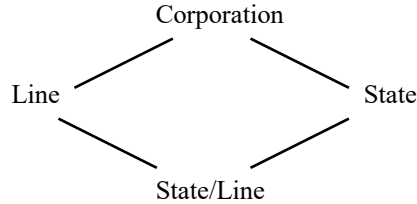
as required, since

$$\begin{aligned}
\sum_l \beta_l^* &= \sum_l \text{Cov}(L_l, L) / \text{Var}(L) \\
&= \text{Cov}(\sum_l L_l, L) / \text{Var}(L) \\
&= \text{Var}(L) / \text{Var}(L) \\
&= 1.
\end{aligned}$$

Notice that the sum of the allocated surplus is irrelevant.

This method of computing premium margins has the desirable property that it allows premium margins to be computed in steps. For example, the corporation could set SBU margins, then the SBU's could take these and compute SBU by line margins. Doing this would give the same answer as going directly to an SBU by line margin, or going first to line and then line/SBU. We will refer to

this as the commutative property of computing margins, because it does not matter which way we "commute" around the diagram below to get to a state/line margin.



The commutative feature is very useful, and would not be present in a model that allocated surplus, owing to the non-additive nature of most surplus allocation regimes. In order to show how it works we need to introduce a new beta:

$$\beta_{xy}^x = \frac{\text{Cov}(r_{xy}, r)}{\text{Cov}(r_x, r)}$$

$$= \frac{\beta_{xy}}{\beta_x}$$

where  $x$  denotes a line and  $xy$  denotes a subline of line  $x$ , for example homeowners and homeowners in New York. If  $x$  is the whole corporation  $r_x=r$  and so the modified beta equals the usual beta. If  $y$  is the whole of line  $x$ ,  $r_x=r_{xy}$  and so again the modified beta equals the usual beta.

This definition shows that to compute a margin for line  $xy$  it is still necessary to compare  $xy$  with  $r$ , that is, it is always necessary to go back to the top level. This is because while subline  $xy$  of line  $x$  might be uncorrelated with line  $x$ , it might be correlated with other lines in the corporation. For example, suppose a company writes homeowners in Florida and workers comp in California and Florida. The company is now considering writing homeowners in California and needs a premium margin. If  $x$  is homeowners and  $xy$  is homeowners in California, then the return on homeowners in California will appear uncorrelated with the existing Florida homeowners book and so will get a low beta. However, California homeowners might be more highly correlated with workers comp in California (through the earthquake hazard) and so require a higher margin. The moral is that you have to look at the whole risk portfolio to set premium margins; it cannot be done in isolation from the other policies written by the corporation.

Given that we have computed a target rate of return for  $x$  we can compute a target rate of return for  $xy$  as

$$Er_{xy} = r_f + \beta_{xy}^x (Er_x - r_f).$$

It now follows easily that premium margins are commutative:

$$\begin{aligned}
\pi_{xy} &= \beta_{xy} (Er - r_f) S_{xy} / P_{xy} \\
&= \frac{\beta_{xy}}{\beta_x} \beta_x (Er - r_f) S_{xy} / P_{xy} \\
&= \frac{\beta_{xy}}{\beta_x} (Er_x - r_f) S_{xy} / P_{xy} \\
&= \beta_{xy}^x (Er_x - r_f) S_{xy} / P_{xy}.
\end{aligned}$$

The first row is computing the margin from top to bottom, the last row is computing from  $x$  to  $xy$ .

### Example

In this section we use the method presented above to compute premium margins by line of business for the whole industry.

The industry example uses Bests loss ratios 1980-93 to derive losses by line using the 1994 premium distribution. Beta stars are then computed from these losses. Premium margins are computed assuming the industry writes at 2:1, targets a 15% return on equity and earns a 5% risk free rate. The resulting margins are shown in Table 1 below, with the lines sorted by increasing margin. The overall margin is, of course, 5%. This calculation is similar to that performed in [2], and the results are consistent.

**Table 1.**  
**Industry Premium Margins by Line of Business**

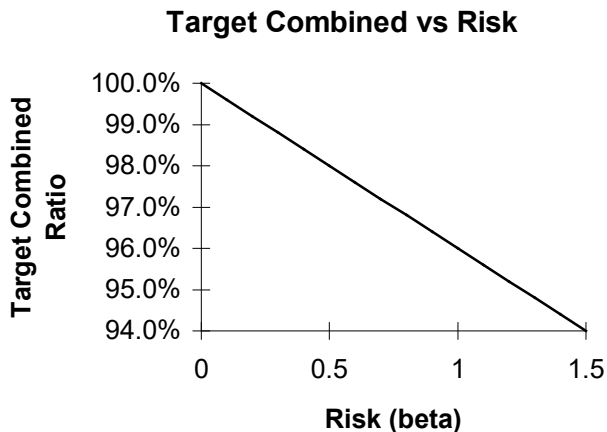
LINE	1993 EP	BETA*	MARGIN	COMBINED
B&THEFT	109,235	0.000	-3.4%	103.5%
OTHER	2,898,031	-0.004	-1.7%	101.7%
GAH	4,258,282	-0.003	-0.8%	100.8%
FARM	1,063,612	0.000	-0.3%	100.3%
OAH	2,381,121	-0.001	-0.3%	100.3%
PPPD	33,562,357	0.013	0.4%	99.6%
OM	1,424,888	0.001	0.6%	99.4%
IM	4,436,596	0.006	1.6%	98.4%
PPAL	57,861,722	0.106	2.2%	97.9%
CAPP	4,303,526	0.010	2.7%	97.3%
FIDELITY	876,114	0.003	3.9%	96.3%
SURETY	2,811,513	0.010	4.0%	96.2%
AIRCRAFT	638,926	0.002	4.4%	95.8%
FIRE	4,377,709	0.018	4.9%	95.4%
WC	30,306,723	0.125	4.9%	95.4%
TOTAL	235,514,167	1.000	5.0%	95.2%
CAL	11,872,692	0.066	6.6%	93.8%
ALLIED	3,168,512	0.019	7.0%	93.5%
BOILER	718,375	0.004	7.2%	93.3%
MM	4,248,596	0.033	9.2%	91.6%
HOMP	20,791,374	0.167	9.5%	91.4%
REINS	9,602,450	0.081	9.9%	91.0%
CMP	16,796,665	0.159	11.2%	89.9%
OL	17,005,148	0.185	12.8%	88.6%

The ranking of lines in Table 2 seems very reasonable, with property damage coverages towards the top and longer tailed liability lines towards the bottom. Homeowners multiperil comes out near the bottom because it is a very large line with very variable results. As Andrew demonstrated, one bad year in homeowners has an impact on the entire industry's results; this type of effect is exactly what beta measures. The analysis is very coarse, using calendar year loss ratios instead of accident year, and ignoring investment income. Including investment income will make the longer tailed lines look worse, since they will become more correlated through their mutual dependence on interest rates.

### Managing the Catastrophe Risk

Beta, as defined above, can be used as part of an overall cat management strategy. Cat and non-cat losses are largely independent, so their beta's can be computed separately and combined. The beta for cat risks measures size of exposure and geographical concentration. It is a subtle enough measure of risk to distinguish between commercial, multi-site risks with identical cat loss distributions (all moments) but different correlations with remaining risks, and it gives a meaningful ranking of risks from most risky to least risky. This ranking is better than just using variance of loss distribution or some notion of geographical PML.

The method given in the first part of the paper allows us to construct a target premium margin, or operating result, for each level of risk. Since the underlying, non-cat risk, is largely common to all risks in a given line the beta method of computing premium margins can be regarded as giving cat risk margin for each line, or policy. For each SBU a graph like the one below would be constructed.



A branch, region or business unit would then look at where it was positioned on this chart. If it is at a risk/combined ratio point above the line then it is bearing too much risk relative to the return it is offering. In this case the branch either needs to decrease the risk, increase the premium (to decrease the combined ratio), or a combination of the two. If increasing the premium to the indicated level is not possible in the market then cat modeling software could be used to evaluate different ways to decrease the risk: percentage windstorm deductibles, policy or agent cancellations, coverage changes, and so on. If the risk/combined ratio point is below the line then premiums could be lowered or more risk undertaken. This might be the case when market forces had severely restricted the market in coastal areas.

When, the branch determines that it cannot possibly raise premiums, it has to decrease exposure. An individual company's required premium margin on a given line of business depends on the distribution of our other exposures; in a case where we have a concentration of exposures our premium margin will be prohibitively high relative to our competitors and we will lose business. This is the market mechanism that forces proper diversification onto insurance



companies, since the insured can move to a better diversified, lower total cost company.

There is a trade-off between cat risk margin and other components of the expense ratio, so that a very low expense company can afford to be less well diversified. This will partly explain how regionals can be successful: they require a higher premium margin to compensate for inadequate diversification (generally the higher premium margin is called "cost of reinsurance"), but offset the additional expense through lower underwriting, processing and loss adjustment expenses, and better knowledge of the markets they serve.

It is instructive to consider what happens as the corporation faces riskier and riskier exposures. There are two dimensions to analyse. Recall our assumption that a corporate profit target  $\Pi$  is given. This target is set by considering the riskiness of the corporation's whole portfolio of risks. The other dimension is allocating  $\Pi$  between lines. If one line  $l$  has a very high variance but is largely uncorrelated with other lines (for example homeowners in Florida for a company that write workers comp on the West coast plus Florida homes) then that line will be required to earn more and more of the profit dollars. We can see this as follows:

$$\begin{aligned}\beta_x^* &= \frac{\text{Cov}(L_x, L)}{\text{Var}(L)} \\ &= \frac{\text{Var}(L_x) + \sum_{y \neq x} \text{Cov}(L_x, L_y)}{\text{Var}(L)} \\ &\approx \frac{\text{Var}(L_x)}{\text{Var}(L_x) + \sum_{y \neq x} \text{Var}(L_y)} \\ &\approx 1\end{aligned}$$

because  $x$  is uncorrelated with other lines and because its variance is assumed to be large relative to that of the other lines. Therefore

$$\begin{aligned}\pi_x &= \frac{S}{P_x} \beta_x^* (Er - r_f) \\ &\approx \frac{S}{P_x} (Er - r_f) \\ &= \frac{\Pi}{P_x}.\end{aligned}$$

In such an unbalanced situation, the model does not declare that line  $x$  cannot be written. Instead, it requires that the profit margin must be very high so line is treated equitably with respect to all other lines. Whether or not such risks are written in the market depends upon the availability of other coverage and on the insured's risk aversion.

If the corporation has set its overall return target after analyzing all the risks currently on the books, and if it is happy the target is adequate, then there is no

need to cancel any risks in order to manage the catastrophe risk; it is only necessary to charge each risk the appropriate gross premium. The gross premium may be uncompetitive in the market and so business will be lost. Where this happens, the corporation was over-exposed relative to its competitors, and market forces are merely working to ensure a more efficient allocation of exposures between companies.

## **Bibliography**

- [1] Bass, I.K. and Khury, C.K., "Surplus Allocation: An Oxymoron" *Insurer Financial Solvency*, CAS Discussion Paper Program, 1992
- [2] Feldblum, Sholom. "Risk Loads for Insurers," *CAS Proceedings*, 1990, Vol LXXVII
- [3] Robbin, Ira, "The Underwriting Profit Provision", Study Note for CAS Exam Part 6
- [4] Miccolis, R.S., "An investigation of Methods, Assumptions, and Risk Modeling for the Valuation of Property-Casualty Insurance Companies" *Financial Analysis of Insurance Companies*, CAS Discussion Paper Program, 1987
- [5] Ingersoll Jr, Jonathan E., *Theory of Financial Decision Making*, Rowman & Littlefield, 1987